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Transverse acoustic soliton in anisotropic paramagnetic crystal

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Abstract

We derive a new integrable system of evolutions describing the dynamics of the transverse strain waves propagating in a bulk crystal with ion impurities possessing the spin-$1/2$. This system describes the evolution of the picosecond acoustic pulses corresponding to 'a few cycle pulses'. The Lax representation for this system and its particular cases are presented. Soliton solutions corresponding to the common and particular cases are found.

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1. Introduction

The existence of different physical mechanisms for opto-acoustical interaction enables the generation of acoustical pulses of picosecond duration [1–3]. Since the early 1970s there has been work with electromagnetic-pulse acoustic self-induced transparency (ASIT) on paramagnetic impurities [4, 5]. A set of papers have recently been devoted to the experimental and theoretical investigation of spin–phonon avalanches in paramagnetic crystal with spin-$1/2$ impurities; see, for example, [6, 7].

The theory of ASIT for a transverse pulse propagating in the direction parallel to magnetic field in a spin system with $S = 1/2$ was developed, for example, in [5, 8, 9]. In this theory, the equations describing the acoustic-pulse dynamics under a number of simplifying assumptions were reduced to both nonintegrable and well-known simple integrable models. For the picosecond acoustic pulses, slow envelope approximation cannot be applied as a rule. Therefore, an analogy with conventional theory of optical self-induced transparency for quasi-monochromatic waves does not apply.

2. Derivation of model

The main purpose of this paper is the study of the new transverse acoustic-pulse dynamics in anisotropic paramagnetic crystal. More precisely, we consider anisotropic interaction of the transverse sound waves and spin system. At the same time we assume that phase velocities of the components of this sound wave propagating in a one-dimensional medium are equal to each other. This situation takes place in elastically isotropic crystals, such as alkali metal halides with central interaction forces between the atoms [10].

For this purpose, a new general integrable model that describes the one-dimensional dynamics of pulses propagating in a crystal with paramagnetic impurities with spin $S = 1/2$ is proposed without using the slow envelope approximation for amplitudes of the pulses. We show that the inverse scattering transform method (ISTM) [11] can be applied to this system and we derive integral Marchenko equations.

To derive a physical model we consider a crystal that is at the temperature of liquid helium. Since the pulse is very short, we also neglect the relaxation terms in the equations for the spin movement.

We assume that a homogeneous static external magnetic field B is directed along the z-axis. Let us suppose a strain pulse propagates along the z-axis. We consider only pulses of lateral strain in a crystal which has a symmetry that belongs to the rhombic system [12]. The Hamiltonian of the system 'strain plus paramagnetic impurities' can then be written in the form

$$H = \int \left\{ n\beta_0 \mathbf{B}_0 \cdot \hat{g}\mathbf{S} + \frac{1}{2\rho_0}\left(P_x^2 + P_y^2\right) + \frac{\rho_0}{2}\left[\alpha_x^2\left(\frac{\partial u_x}{\partial z}\right)^2 + \alpha_y^2\left(\frac{\partial u_y}{\partial z}\right)^2\right]\right\} \mathrm{d}\mathbf{r} \tag{1}$$

where ρ_0 is the average medium density, u_x and u_y are the components of lateral displacements in solids, $P_{x(y)} = \rho_0 \partial u_{x(y)}/\partial t$, n is the concentration of paramagnetic centres, $\alpha_{x(y)}$ is the velocity of the lateral strain wave at the displacement in the $x(y)$ direction in the absence of paramagnetic impurities, β_0 is the Bohr magneton, $\mathbf{S} = (\hat{\sigma}_x, \hat{\sigma}_y, \hat{\sigma}_z)$ is the effective spin of s ion impurity, and $\hat{g}$ is the Landé tensor. We assume that $S = 1/2$ and $\hat{\sigma}_\gamma$ are the Pauli matrices. The constants v_x and v_y are related to the elastic moduli of a crystal [12]: $v_x^2 = c_{55}/\rho_0 = \alpha_x^2/2$, $v_y^2 = c_{44}/\rho_0 = \alpha_y^2/2$. The axes of x, y and z are perpendicular to the planes of crystal symmetry. The Landé tensor can be written in the form

$$\hat{g} = \hat{g}_0 + \delta\hat{g} \tag{2}$$

where $\hat{g}_0$ is the Landé tensor in the absence of crystal strain such that

$$\mathbf{B}_0 \cdot \hat{g}_0 \mathbf{S} = B_0 g_{zz} \hat{\sigma}_z \tag{3}$$

and $\delta\hat{g}$ is disturbance due to the strain pulse. For lateral strain pulses, we have (see, for example, [5])

$$\mathbf{B}_0 \cdot \delta\hat{g}\,\mathbf{S} = B_0 g_{zz}(F_x \hat{\sigma}_x \partial u_x/\partial_z + F_x \hat{\sigma}_x \partial u_x/\partial_z) \tag{4}$$

where $F_x = F_{xzxz}$, $F_y = F_{yzyz}$ are the components of spin–strain interaction. Using the Hamiltonian formalism with respect to equations (1)–(4) gives

$$\partial u_{x(y)}/\partial t = \delta H/\delta P_{x(y)} \qquad \partial P_{x(y)}/\partial t = -\delta H/\delta u_{x(y)}. \tag{5}$$

We denote the components of the stress tensor

$$\mathcal{E}_{xz} = \frac{\partial u_x}{\partial z} \qquad \mathcal{E}_{yz} = \frac{\partial u_y}{\partial z}.$$

We use here a quasi-classical description of the spin–phonon interaction, i.e. acoustic fields (components of the strain tensor $\mathcal{E}_{x(y)}$) are classical but the spin system is treated as a quantum system. In this approximation, operators describing the spin system couple to each other only by means of a propagating acoustic field. In such a case, quantum averaging means that we replace a spin operator to a classical scalar component of the vector $\mathbf{S} = (S_x, S_y, S_z)$. After this quantum averaging using equation (5), we find the following evolution equations

$$\frac{\partial^2 \mathcal{E}_{xz}}{\partial t^2} - v_x^2 \frac{\partial^2 \mathcal{E}_{xz}}{\partial z^2} = q_x \frac{\partial^2 S_x}{\partial z^2} \tag{6}$$

$$\frac{\partial^2 \mathcal{E}_{yz}}{\partial t^2} - v_y^2 \frac{\partial^2 \mathcal{E}_{yz}}{\partial z^2} = q_y \frac{\partial^2 S_y}{\partial z^2} \tag{7}$$

where $q_{x(y)} = n\hbar\omega_0 F_{x(y)}/\rho_0$, $\omega_0 = g_{zz}\beta_0 B_0/\hbar$, $\hbar$ is the Plank constant, $S_\gamma = \mathrm{Tr}\,\hat{\sigma}_\gamma \hat{\rho}/2$,

$$S_z = \frac{1}{2}(\rho_{11} - \rho_{22}) \qquad S_x = \frac{1}{2}(\rho_{12} + \rho_{21}) \qquad S_y = \frac{\mathrm{i}}{2}(\rho_{12} - \rho_{21})$$

and $S_z^2 + S_x^2 + S_y^2 = 1$. $\hat{\rho}$ is the density matrix of a two-level transition between the Zeeman level. For the density matrix we have the Heisenberg equations

$$\mathrm{i}\hbar \frac{\partial \hat{\rho}}{\partial t} = [\hat{H}, \hat{\rho}]. \tag{8}$$

Using equation (8) we derive the following Bloch equations for the spin **S** components

$$\frac{\partial}{\partial t} S_x = -\omega_0 S_y + \omega_0 f_y \mathcal{E}_y S_z \tag{9}$$

$$\frac{\partial}{\partial t} S_y = \omega_0 S_x - \omega_0 f_x \mathcal{E}_x S_z \tag{10}$$

$$\frac{\partial}{\partial t} S_z = \omega_0 (f_x \mathcal{E}_x S_y - f_y \mathcal{E}_y S_x) \tag{11}$$

where $f_{x(y)} = F_{x(y)}\hbar^{-1}\omega_0^{-1}$.

To find an integrable reduction of the model we have to impose a few physical restrictions. As pointed out already, we neglect relaxations and assume that the phase velocities of waves are equal: $v_x = v_y = v$. In real media, the concentration of paramagnetic impurities can often be considered small. Then, it is possible to apply the unidirectional propagation approximation similar to that used in [13] for deriving the reduced Maxwell–Bloch equations for a two-level optical medium. In this case, we can write the following formal approximate equality, $\partial_z = -v\partial_t + \mathcal{O}(\epsilon)$, where ϵ is a small parameter. The normalized impurity concentration is of the same order of smallness as the derivative $\partial_z + v^{-1}\partial_t$ of the acoustic field amplitudes. The derivative with respect to z on the right-hand sides of equations (6) and (7) can be replaced by $v^{-1}\partial_t$ with an accuracy of $\mathcal{O}(\epsilon^2)$.

Using the above approximations and taking the moving system of coordinates, we derive the following new integrable system of evolution equations

$$\begin{aligned} &\frac{\partial S_x}{\partial \tau'} = -S_y + E_y S_z \qquad \frac{\partial S_y}{\partial \tau'} = S_x - E_x S_z \qquad \frac{\partial S_z}{\partial \tau'} = E_x S_y - E_y S_x \\ &\frac{\partial E_x}{\partial \chi} = \frac{\partial S_x}{\partial \tau'} \qquad \frac{\partial E_y}{\partial \chi} = r^2 \frac{\partial S_y}{\partial \tau'} \end{aligned} \tag{12}$$

where $r = F_y/F_x$, $E_{x(y)} = f_{x(y)}\mathcal{E}_{x(y)}$, $\tau' = \omega_0(t - v^{-1}z)$, $\chi = zq_x F_x(2v\hbar)^{-1}$.

3. The ISTM technique

The Lax representation for system (12) has the following compact form

$$\partial_{\tau'}\Phi = \frac{1}{2}\begin{pmatrix} -\mathrm{i}\,\mathrm{cn}\,\mathrm{dn} & \mathrm{dn}\,E_x - \mathrm{i}\,\mathrm{cn}\,E_y \\ -\mathrm{dn}\,E_x - \mathrm{i}\,\mathrm{cn}\,E_y & \mathrm{i}\,\mathrm{cn}\,\mathrm{dn} \end{pmatrix}\Phi \equiv \hat{L}\Phi \tag{13}$$

$$\partial_\chi \Phi = \frac{1}{2\mathrm{sn}^2}\begin{pmatrix} -\mathrm{i}\,\mathrm{cn}\,\mathrm{dn}\,S_z & \mathrm{dn}\,S_x - \mathrm{i}\,\mathrm{cn}\,S_y \\ -\mathrm{dn}\,S_x - \mathrm{i}\,\mathrm{cn}\,S_y & \mathrm{i}\,\mathrm{cn}\,\mathrm{dn}\,S_z \end{pmatrix}\Phi \equiv \hat{A}\Phi. \tag{14}$$

Here we use the parametrization by the Jacobi elliptic functions, $\mathrm{sn} = \mathrm{sn}(\zeta, r), \mathrm{cn} = \mathrm{cn}(\zeta, r) = \sqrt{1-\mathrm{sn}^2}, \mathrm{dn} = \mathrm{dn}(\zeta, r) = \sqrt{1-r^2\mathrm{sn}^2}$, where ζ is a spectral parameter, and r is the modulus of the Jacobi functions.

The Lax presentation of system (12) admits the algebraic forms of dependence of the spectral parameter for the nonoverlapping r values: $|r| = 1$, for $r = 0$, and for $|r| \neq 1, r \neq 0$.

In the isotropic case, $|r| = 1$, the Lax pair (13) and (14) reduces to the form

$$\partial_{\tau'}\Phi = \frac{1}{2}\begin{pmatrix} -\mathrm{i}\xi^2 & \xi(E_x - \mathrm{i}E_y) \\ -\xi(E_x + \mathrm{i}E_y) & \mathrm{i}\xi^2 \end{pmatrix}\Phi \equiv \hat{L}_1\Phi \tag{15}$$

$$\partial_{\chi}\Phi = \frac{1}{2(1-\xi^2)}\begin{pmatrix} -\mathrm{i}\xi^2 S_z & \xi(S_x - \mathrm{i}S_y) \\ -\xi(S_x + \mathrm{i}S_y) & \xi^2 S_z \end{pmatrix}\Phi \equiv \hat{A}_1\Phi \tag{16}$$

where ξ is the new spectral parameter. The spectral problem (15) is of the Kaup–Newell type [14].

For the limit of the strong anisotropy of interaction, $r = 0$, we derive from equations (13) and (14) the following Lax pair

$$\partial_{\tau'}\Phi = \frac{1}{2}\begin{pmatrix} -\mathrm{i}\lambda & E_x - \mathrm{i}\lambda E_y \\ -E_x - \mathrm{i}\lambda E_y & \mathrm{i}\lambda \end{pmatrix}\Phi \equiv \hat{L}_0\Phi \tag{17}$$

$$\partial_{\chi}\Phi = \frac{1}{2(1-\lambda^2)}\begin{pmatrix} -\mathrm{i}\lambda S_z & S_x - \mathrm{i}\lambda S_y \\ -S_x - \mathrm{i}\lambda S_y & \lambda S_z \end{pmatrix}\Phi \equiv \hat{A}_0\Phi \tag{18}$$

where λ is the new spectral parameter and E_y is now an arbitrary real function of the variable τ'. Physically, such a situation may correspond to a case when a short acoustic pulse E_x polarized along the x-axis propagates in the positive direction along the z-axis. An additional acoustic field with the amplitude E_y propagates along the z-axis in the opposite direction. It can be shown that in a coordinate system moving with the pulse E_x we can neglect the dependence of the field E_y on χ with an accuracy $\mathcal{O}(\epsilon)$. Therefore, E_y is a function of τ' only.

Under the assumptions that $E_y(\tau') \equiv \mathrm{const}$ and $r = 0$, system (12) can be transformed to the equations describing unidirectional propagation of electromagnetic pulses in a two-level media. Indeed, we denote

$$R_x = S_x \qquad R_y = \frac{S_y - E_y S_z}{\Omega} \qquad R_z = \frac{E_y S_y + S_z}{\Omega} \tag{19}$$

where $\Omega = \sqrt{1+E_y^2}$, then system (12) for the components of the vector $\mathbf{R} = (R_x, R_y, R_z)$ and the field E_x becomes

$$\partial_{\tau'}R_x = -R_y\Omega \tag{20}$$

$$\partial_{\tau'}R_y = R_x\Omega - E_x R_z \tag{21}$$

$$\partial_{\tau'}R_z = E_x R_y \tag{22}$$

$$\partial_{\chi}E_x = -\Omega R_y. \tag{23}$$

System (20)–(23) is formally equivalent to the reduced Maxwell–Bloch equations [13] describing interaction of a linearly polarized light pulse with a nondegenerate two-level system.

Thus, the spectral problem (13) includes as particular cases the Kaup–Newell spectral problem and the Zakharov–Shabat spectral problem for real potential.

Let us return to the main integrable system (12). Consider the meanings of parameter r that $|r| \neq 1$ and $r \neq 0$. For this case, we are able to derive an algebraic parametrization of the

Lax pair for system (12) introducing the new spectral parameter ξ by the following equivalent relations:

$$\mathrm{cn}(\zeta, r) = \frac{\sqrt{1-r^2}}{2r}\left(\xi - \frac{1}{\xi}\right) \qquad \mathrm{dn}(\zeta, r) = \frac{\sqrt{1-r^2}}{2}\left(\xi + \frac{1}{\xi}\right). \tag{24}$$

Then the Lax pair (13) and (14) for system (12) becomes

$$\partial_\tau \Phi = \begin{pmatrix} -\mathrm{i}\left(\xi^2 - \frac{1}{\xi^2}\right) & \xi E^* + \frac{1}{\xi}E \\ -\xi E - \frac{1}{\xi}E^* & \mathrm{i}\left(\xi^2 - \frac{1}{\xi^2}\right) \end{pmatrix} \Phi \tag{25}$$

$$\partial_\chi \Phi = \frac{2r^2(1-r^2)^{3/2}\xi^2}{4r^2 - [(1-r^2)\xi^2 - 1 - r^2]^2} \begin{pmatrix} -\mathrm{i}\frac{a}{r}\left(\xi^2 - \frac{1}{\xi^2}\right)S_z & \xi S^* + \frac{1}{\xi}S \\ -\xi S - \frac{1}{\xi}S^* & \mathrm{i}\frac{a}{r}\left(\xi^2 - \frac{1}{\xi^2}\right)S_z \end{pmatrix} \Phi \equiv \hat{A}\Phi \tag{26}$$

where $a = \sqrt{1-r^2}/2$, $E = rE_x/a + \mathrm{i}E_y/a$, $S = S_x + \mathrm{i}S_y/r$, and $\tau = a^2\tau'/(2r)$. The spectral problem (13) is a new one to our knowledge.

In the theory of integrable systems, the spectral problems (13) and (25), likely, arise for the first time. However, the corresponding ISTM apparatus has much in common with the apparatus developed earlier for the related spectral problems arising when solving the Thirring equation [15], and the nonlinear differential Schödinger equation [14]. For this reason, we discuss only the key elements of the ISP apparatus for a potential E that decreases sufficiently fast at infinity.

The spectral problem (25) possesses the following symmetry properties

$$\Phi = \hat{M}\Phi(\xi^*)^*\hat{M}^{-1} \tag{27}$$

where

$$\hat{M} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \tag{28}$$

and

$$\Phi(\xi^*)^* = \Phi(\xi^{-1}). \tag{29}$$

A potential $E(\tau, \chi)$ rapidly vanishes when $\tau \to \pm\infty$. We introduce the Jost functions Φ, the solutions of equation (25) with the asymptotics

$$\Phi^\pm = \exp(-\mathrm{i}\omega\sigma_3\tau) \qquad \tau \to \pm\infty. \tag{30}$$

where $\omega = \xi^2 - \xi^{-2}$.

We introduce the scattering matrix $\hat{T}$ by the formula

$$\Phi^- = \Phi^+\hat{T} \tag{31}$$

where

$$\hat{T} = \begin{pmatrix} a^* & b \\ -b^* & a \end{pmatrix}. \tag{32}$$

The dependence of the scattering data versus χ is governed by the equation

$$\partial_\chi \hat{T} = -\hat{T}\,\mathrm{e}^{-\mathrm{i}\sigma_3\omega\tau}\hat{A}(\tau = -\infty)\,\mathrm{e}^{\mathrm{i}\sigma_3\omega\tau} + \mathrm{e}^{-\mathrm{i}\sigma_3\omega\tau}\hat{A}(\tau = \infty)\,\mathrm{e}^{\mathrm{i}\sigma_3\omega\tau}\hat{T}. \tag{33}$$

We present the Jost function in the integral form

$$\Phi^+(\tau) = \mathrm{e}^{-\mathrm{i}\hat{\sigma}_3[\omega\tau + \mu(\tau)]} + \int_\tau^\infty \begin{pmatrix} [Q_1(\tau,s) + \xi^{-2}Q_2(\tau,s)]\,\mathrm{e}^{-\mathrm{i}\mu(\tau)} & -[\xi K_1^*(\tau,s) + \xi^{-1}K_2(\tau,s)]\,\mathrm{e}^{-\mathrm{i}\mu(\tau)} \\ [\xi K_1(\tau,s) + \xi^{-1}K_2^*(\tau,s)]\,\mathrm{e}^{\mathrm{i}\mu(\tau)} & [Q_1^*(\tau,s) + \xi^{-2}Q_2^*(\tau,s)]\,\mathrm{e}^{\mathrm{i}\mu(\tau)} \end{pmatrix} \times \mathrm{e}^{-\mathrm{i}\hat{\sigma}_3\omega s}\,\mathrm{d}s \tag{34}$$

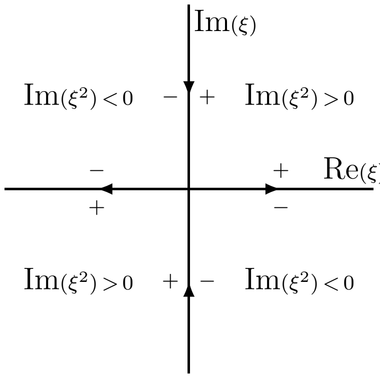

Figure 1. The counterclockwise contour Γ includes the paths along the axes as shown by the arrows. The contour encloses the first and third quadrants of $\mathrm{Im}\,\xi^2 \geqq 0$, and that passes above all poles in the first quadrant and below all poles in the third quadrant.

where $\hat{\sigma}_3$ is the Pauli matrix and $\mu(\tau)$ is a real function, which will be determined below. From equations (25) and (34) it follows that

$$E(\tau,\chi) = -2K_2(\tau,\tau,\chi)\exp(-2\mathrm{i}\mu) \tag{35}$$

$$E^*(\tau,\chi) = -2K_1^*(\tau,\tau,\chi)\exp(-2\mathrm{i}\mu). \tag{36}$$

In order for equation (34) to be valid, it is necessary that

$$\lim_{s\to\infty} K_{1,2}(\tau,s) = 0 \tag{37}$$

$$\lim_{s\to\infty} Q_{1,2}(\tau,s) = 0. \tag{38}$$

The following identities are valid

$$\int_\Gamma \xi^m \exp(\mathrm{i}\omega\tau)\,\mathrm{d}\xi = 4\pi\delta(\tau) \qquad m = 1, -3 \tag{39}$$

$$\int_\Gamma \xi^{2m} \exp(\mathrm{i}\omega\tau)\,\mathrm{d}\xi = 0 \qquad m = -1/2, 0, \pm1, \pm2, \ldots \tag{40}$$

where the contour of integration is shown in figure 1. Using equations (39), (40) and (34) we derive from equation (31) the following Marchenko equations

$$K_1^*(\tau,y) = F_0(\tau+y) + \int_\tau^\infty [Q_1(\tau,s)F_0(s+y) + Q_2(\tau,s)F_{-1}(s+y)]\,\mathrm{d}s \tag{41}$$

$$K_2(\tau,y) = F_{-1}(\tau+y) + \int_\tau^\infty [Q_1(\tau,s)F_{-1}(s+y) + Q_2(\tau,s)F_{-2}(s+y)]\,\mathrm{d}s \tag{42}$$

$$Q_1^*(\tau,y) = -\int_\tau^\infty [K_1(\tau,s)F_1(s+y) + K_2^*(\tau,s)F_0(s+y)]\,\mathrm{d}s \tag{43}$$

$$Q_2^*(\tau,y) = -\int_\tau^\infty [K_1(\tau,s)F_0(s+y) + K_2^*(\tau,s)F_{-1}(s+y)]\,\mathrm{d}s \tag{44}$$

where $y \geqslant \tau$. The kernel F has the form

$$F_m(y,\chi) = \int_{-\infty}^\infty \frac{b(\chi)}{a(\chi)}\frac{\xi^{2m}\,\mathrm{e}^{-\mathrm{i}\omega y}}{2\pi}\,\mathrm{d}\xi - \mathrm{i}\sum_k \frac{\xi_k^{2m}c_k(\chi)\,\mathrm{e}^{-\mathrm{i}\omega_k y}}{a'(\xi_k,\chi)} \tag{45}$$

where $\omega_k = \xi_k^2 - \xi_k^{-2}$.

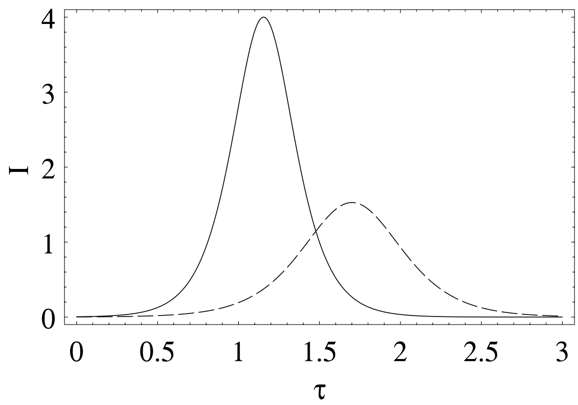

Figure 2. Intensity of solitons $I = |E|^2$ versus τ for $\lambda_1 = \exp(\mathrm{i}\phi_1)$, $\phi_1 = \pi/3$ (solid line) and $\phi_1 = \pi/5$ (dashed line).

The simplest solution corresponds to a single pole ξ_1 lying in the first or third quadrants of the complex plane and the conditions, $E(\pm\infty, \chi) = 0$, $S_z(0, \chi) = -1$. Owing to the symmetry property (29) ξ_1 obeys the condition $|\xi_1| = 1$. Let $\xi_1 = \exp(\mathrm{i}\phi_1)$, where $\phi_1 \in \mathbb{R}$. The one-pole solution can be derived by solving the system (33), (41) and (45). This solution is

$$E(\tau, \chi) = \frac{-2|\sin\phi_1| \exp[\mathrm{i}\gamma_1 - \mathrm{i}\phi_1/2]}{|\cosh[4\sin\phi_1\theta + \gamma_2 - \mathrm{i}\phi_1/2]|} \tag{46}$$

where

$$\theta = \tau - \frac{a\chi}{2\sqrt{r^2\cos^2(\phi_1/2) + \sin^2(\phi_1/2)}} \qquad \gamma_2 = \ln\left|\frac{c_1}{2a'(\xi_1)}\right|$$

and $\gamma_1 = \arg[(-\mathrm{i}c_1)/(2a'(\xi_1))] = N\pi$, $N = 0, 1$, due to symmetry (29). The soliton intensity for different ϕ_1 is shown in figure 2.

Note that for the two-soliton solution corresponding to a pair of poles $\zeta_1 = \eta$, $\zeta_2 = 1/\eta$ the symmetry property (29) does not lead to restrictions such as $|\eta| = 1$.

4. Conclusion

We find an integrable system of evolution equations describing the dynamics of acoustic fields in paramagnetic crystal with spin-1/2 impurities. This system can be used for investigation of acoustic pulse formation propagating in a crystal with a rhombic symmetry. Evolution equations are derived under the assumption of the unidirectional propagation of pulses and without using the approximation of slow changing amplitudes and phases of acoustic fields.

We estimate the parameters of fields and the medium required for observation of formation of the acoustic picosecond pulses. Consider, for instance, a crystal of MgO containing paramagnetic impurities Fe^{2+} at the temperature $T = 4$ K. Let the magnetic field strength be such that the Zeeman splitting is $\omega_0 = 10^{12}$ s^{-1}. This corresponds to the realistic strength of the magnetic field. Coefficients of the medium are as follows [12]: $G_\gamma \sim 10^{-13}$ erg, $n \sim 10^{19}$ cm^{-3}, $n_0 \sim 3 - 4$ g cm^{-3}, $v \approx 5 \times 10^5$–10^6 cm s^{-1}, $\lambda_\gamma \approx 5 \times 10^{11}$–$10^{12}$ din cm^{-2}. Under such conditions the peak intensity of the acoustic pulse can be $I \sim 10^8$ V cm^{-2} and the duration can be $\tau_p \sim 10$ ps.

Recently, the observation of the phonon avalanche has been described in [6, 7]. The authors of these papers have explored the nonsolitonic regime of spin–sound interaction, which is analogous to the superradiance phenomena in nonlinear optics. Their theoretical treatment of the observed effects was based on using the Maxwell–Bloch equations describing quasi-monochromatic pulse dynamics in a two-level medium. The conditions of these experiments are available for observation of the unidirectional soliton formation of picosecond duration in the system considered in the present paper. The generation and investigation of the acoustic pulses with picosecond duration in another physical system is described, for example, in [3].

The dynamics of a few-cycle acoustic pulses in a crystal possesses some peculiarities and novel features in comparison with that of quasi-monochromatic pulses. It is known that for picosecond time scale τ_p in some crystals losses associated with sound pulse propagations are proportional to τ_p^{-2} [1]. For quasi-monochromatic pulses with the carrying frequency ω_0 durations are at least of the order $\gtrsim 10\,\omega_0^{-1}$. Therefore, for the same durations losses corresponding to a few-cycle pulses are at least 100 times less than those of a quasi-monochromatic pulse. The second difference that follows from the obtained solution (46) is that the form of soliton essentially depends on the initial phase α_1 (see figure 2) contrary to the case of soliton solution of the sine-Gordon equation or the Maxwell–Bloch equation. The latter two models have been used for the description of dynamics of quasi-monochromatic acoustic pulse evolution equations by the authors of [4, 8, 9]. The initial phase of soliton solutions of these models does not have any influence on the amplitude and duration of a pulse. We reveal that for few-cycle acoustic pulses there are new opportunities for the determination of a pulse phase by controlling its amplitude. This property may be useful for the diagnostic of the medium by using propagating acoustic solitons [1]. Thus, studies of the formation of the picosecond acoustic pulses and the investigation of their dynamics in paramagnetic crystal with spin impurities are of practical interest. Effects related to the anisotropy of interaction of acoustic wave and spin system are revealed in the asymmetrical distribution of energy between the transverse components of the acoustic wave. These components are proportional to the real and imaginary parts of solution (46).

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